Rotation Project : Material Point Method for Simulating Non-Newtonian Fluids

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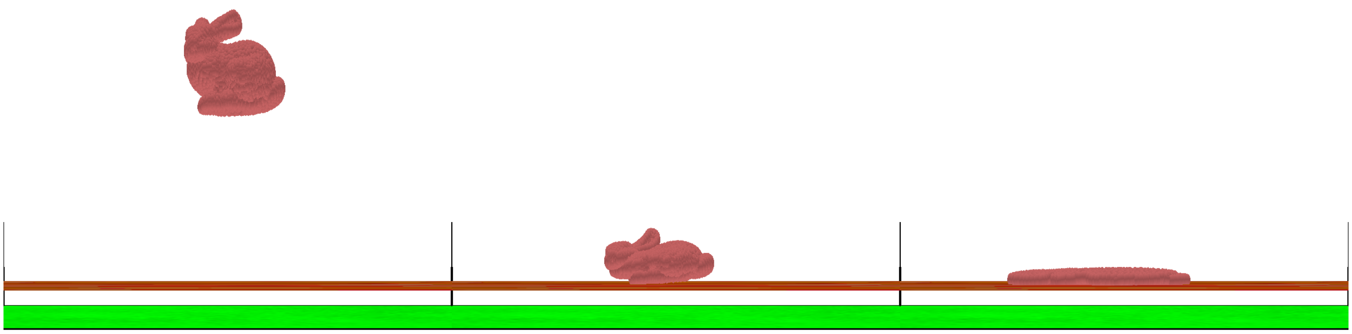


Figure 1: Oobleck bunny dropped to ground jumps before settling.

1 INTRODUCTION

1.1 Motivation

Non-Newtonian fluids are interesting objects. If modelled properly they can behave as a creamy foam, a fresh pie or as an oozing oobleck. They have non-linear behavior, and depending on the amount of stress applied they can behave as a solid or as a liquid. Generally, Lagrangian approaches are best suited for solid materials while Eulerian methods works well for liquid behavior. Thus to simulate fluids which behave as both solids and liquids, material point method is preferred. It effectively combines the advantages of both Eulerian and Lagrangian techniques to simulate a large variety fluids.

In this rotation, we worked on simulating such non-Newtonian fluids using the Herschel-Bulkley model as proposed by [Yue et al. 2015]. We used the MPM formulation as detailed by [Klár et al. 2016]. We later added adaptive time stepping to speed up the computation time to simulate the different objects. Since, the field of fluid simulation was new for me. Thus, I will present this report for a reader who is interested in learning it from start. If you have experience with fluid simulation feel free to skip the initial background sections.

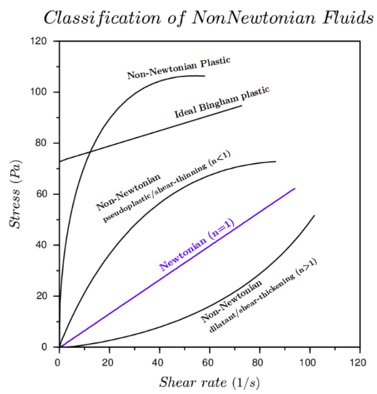
This report is organised as follows: first, we briefly introduce the readers to different types of fluids and different simulation techniques using particle based method namely PIC and FLIP. Later, we discuss the Material Point Method (MPM) as introduced by [Stomakhin et al. 2013]. We then introduce the Herschel-Bulkley constitutive model's temporal discretization as described by [Yue et al. 2015], followed by adaptive time stepping method to speed up the simulation time. Lastly, we conclude our discussion with the results that we obtained during this rotation and potential future work.

2 BACKGROUND : TYPES OF FLUIDS

Fluids are defined into two categories - Newtonian Fluids and Non-Newtonian Fluids. Newtonian fluids follows newton's law of viscosity where the viscosity of the fluid does not change with time. It manifests itself in the fact that stress that the fluid experiences is directly proportional to the amount of strain. Thus, stress and strain have a linear relationship. On the other hand, Non-Newtonian fluids do not follow newton's law of viscosity and thus they show interesting stress-strain relationship. The figure **2a** displays the different fluids types and figure **2b** lists some examples of these categories. For this rotation we focused on shear thickening and shear thinning fluids because of their unique interaction properties with applied force. Shear thickening materials behave as solids when high stress is applied otherwise they flow as liquid. Whereas, shear thinning fluids flow more when additional stress is applied.

3 BACKGROUND : INTRODUCTION TO FLUID SIMULATION

Fluid simulation describes the procedure of generating not only cool animations in virtual world but also help us understand the physics behind these phenomena more properly. The book by [Bridson 2015] gives an excellent introduction to a beginner in the field of Fluid simulation. The **stable fluids** paper by [Stam 1999] is a great next step. It explains the basic concepts with an easy to implement application of Navier-Stokes equation. The simulation method presented in the paper is unconditionally stable and allows the user to use large time steps thus making the simulation faster. But this method damps the fluid motion. Within short time, the



(a) Different types of fluids showcasing different stress/strain relationship. Source - wiki

Type of behavior	Description	Example
Newtonian fluids	Viscosity does not change with time	Water, oil, alcohol
	Non- Newtonian Fluids	
Shear Thickening (dilatant)	Apparent Viscosity increases with increased stress	Corn Starch in water, oobleck
Shear Thinning (pseudoplastic)	Apparent Viscosity decreases with decreased stress	Whipped cream, ketchup, blood, sand in water.
Visco-elastic Materials	Between fluids and solids which displays both viscous and elastic effects	Lubricants, Silly Putty.
	Time Dependent Non-Newtonian Fluids	
Rheopecty	Apparent viscosity increases with the duration of the stress	Gypsum paste, printer ink
Thixotropic	Apparent viscosity decreases with the duration of the stress	Peanut butter, yoghurt.

(b) Examples of fluids in different categories.

fluid simulation settles down and user driven impulse is needed to keep the simulation going.

To avoid the damping issue with stable fluids, Particle in a cell or **PIC** [Harlow and Welch 1965] became popular for fluid simulation. This was a hybrid method between pure Lagrangian and pure Eulerian paradigms and utilizes the best of both the worlds to get stable but undamped fluid simulation. The Lagrangian particles are used to hold the mass, momentum and deformation gradient while the Eulerian grid is used as a scratch pad to perform the dynamics update. The intermediate steps of PIC simulation are as follows -

- **Particles to Grid** : Mass and velocity is transferred from the particles and interpolated on the grid.
- **Grid Dynamics** : The grid forces based on the constitutive equation of the fluid is computed and applied on the grid particles. The grid velocities are updated. All the grid based collision are also resolved in this step.
- **Grid to Particles** : Lagrangian particle information is updated and interpolated back from the grid to the particles. This leads to update of velocity and deformation gradient of the particles. At last, the particle position is updated.

This method tries to improve over stable fluid by making the simulation less dissipative. But even this method cannot prevent rotational motion damping as the transfer of information from grid back to particles does not preserve angular momentum [Jiang et al. 2015]]. This loss of angular momentum results in rotational motion damping.

Because of the rotational damping limitation of the PIC method, researchers came up with an upgrade on PIC - named **FLIP** : Fluid Implicit Particle [Brackbill et al. 1988]. This method is very similar to PIC but includes a special and potentially dangerous step where during the **grid to particle** step instead of interpolating velocities from grid it just updates the previous particle velocity with the interpolation of difference of the grid velocity of current and previous steps. This step allows the FLIP to get rid of damping of rotation motion that PIC suffers from. But because of this dangerous step it creates a numerical instability and leads to a noisy simulation. The following equations explain the difference between the two update

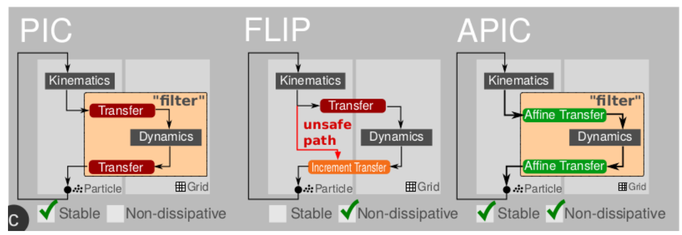


Figure 3: Overview of different particle based methods. Source - [Jiang et al. 2015]

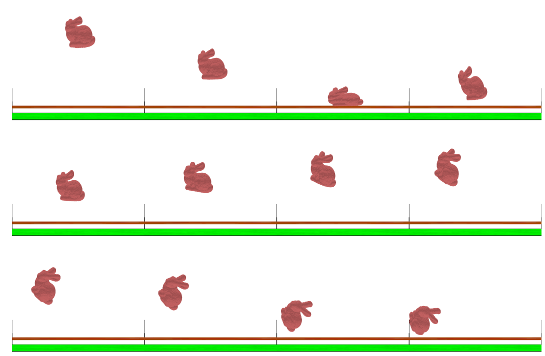


Figure 4: Showcasing the elastic bunny made of pure elastic material in action

steps of PIC and FLIP to present a clearer picture [Stomakhin et al. 2013]

$$PIC : v^{n+1}_{pic(p)} = \sum_{n=0} v_i^{n+1} w_{ip}^n \quad , and$$

$$FLIP : v^{n+1}_{flip(p)} = v_p^n + \sum_{n=0} (v_i^{n+1} - v_i^n) w_{ip}^n .$$

4 BACKGROUND : MATERIAL POINT METHOD

Material Point Method : **MPM** was first introduced by [Sulsky et al. 1995] but it was introduced to the graphics community in the snow

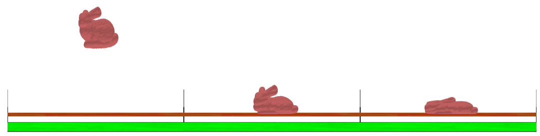


Figure 5: Showcasing the pie made of bunny colliding with ground

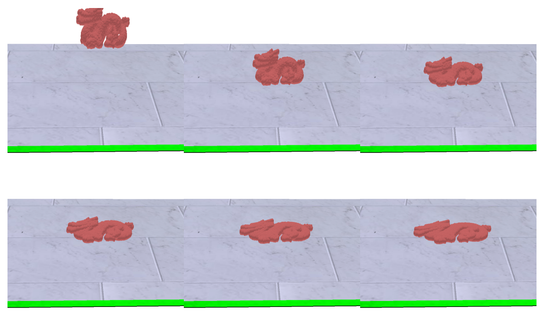


Figure 6: The dragon made of oobleck is dropped from height and then it droops on the ground

simulation paper by [Stomakhin et al. 2013]. MPM is an extension of FLIP and PIC and applies it to increase the range of materials. Using MPM researchers have been successful in simulating snow [Stomakhin et al. 2013], sand [Klár et al. 2016], a large number of non-Newtonian fluids, visco-elastic and visco-plastic materials showcasing shear thickening and shear thinning [Yue et al. 2015], foams and sponges [Ram et al. 2015] [Fang et al. 2019] to name a few. To get the stability of PIC and undamped rotation of FLIP [Stomakhin et al. 2013] used a combination of the two steps to define the velocity update in the grid to particle step. $\alpha \epsilon [0, 1]$ is used as a constant to interpolate between the two velocities. Thus, the final velocity update looks as follows -

$$v_p^{n+1} = (1-\alpha)v_{pic(p)}^{n+1} + \alpha v_{flip(p)}^{n+1} \tag{1}$$

Another important topic in MPM is the **Constitutive Model** used. The constitutive model describes the flow of matter and their unique elastic - plastic response to different deformations in response to an applied force. It defines the behavior of the material and thus, the different constitutive model are best suited for simulating different materials as they capture the material's unique property more properly. For example porous solids like snow and sand follow a variant of Drucker-Prager plasticity model [Stomakhin et al. 2013], [Klár et al. 2016], while Herschel-Bulkley model is used to simulate a large number of non-Newtonian fluids as described in [Yue et al. 2015]. For visco-elastic materials a variation of St. Venant elastic model [Klár et al. 2016] or variation of Neo-Hookean-Piola method as described in [Yue et al. 2015] are popularly used. The good thing about MPM is that every particle in the simulation can be subjected to perform dynamics update based on its own constitutive model and thus MPM can simulate the whole zoo of different material particles.

MPM is able to simulate a large variety of fluids, but it still suffers from significant damping. It is noticed that adding even a small contribution of PIC update in equation **1** leads to significant damping of rotation motion present in the fluid simulation. But without the small contribution of PIC, pure FLIP is just too noisy. To avoid this conundrum [Jiang et al. 2015] proposed an Affine Particle in Cell : **APIC** method which removes the rotation damping altogether and leads to much better realization of fluid simulation. Figure **3** overviews APIC and depicts why it is advantageous over PIC and FLIP.

5 MODIFICATIONS : MATERIAL POINT METHOD

For the purpose of this rotation, we were looking to simulate non-Newtonian fluids with a special focus on fluids which have shear-thickening (eg - corn starch solution in water , oobleck) or shear-thinning (eg - whipped cream, ketchup) properties. We used the formulation as given in [Yue et al. 2015] as our basis to simulate such fluids. Instead of using combination of PIC and FLIP based update, we used the APIC formulation for better performance of the fluid simulation. We implemented the Herschel-Bulkley Temporal Discretization as specified in the paper. It includes **Elastic Prediction** : which increments the deformation gradient. If the gradient deformation exceeds the **Von-Mises** criterion we then apply the plastic correction and the excess deformation is converted to permanently deformed plastic deformation from which the fluid cannot recover its original shape. For detailed explanation refer section 5 of [Yue et al. 2015].

6 ADAPTIVE TIME STEPPING - SPEEDING UP THE COMPUTATION

During the rotation we realized that although the simulations that we generated properly demonstrates the behavior of the respective fluids, the time required to simulate it was high. There were instances where the entire fluid was just translating without undergoing any deformation and thus the time-step can be stepped up. With this in mind, we implemented the Adaptive-time-stepping method to speed the performance of our simulation.

After a fix number of frames the algorithm seeks if it can increase the time step. The feasibility of the update is governed by change in the deformation gradient. If it is within 10% of the previous deformation gradient we update the time step, else we roll back to last stable time step. In addition to this, we also apply the CFL time step restriction and elastic wave time step restriction as further checks on the time step. The minimum amongst the three limit on time-step is used as the final time step for that simulation step. We used the formulations for this restrictions as given in [Fang et al. 2018]. We modified their work to utilize the time stepping criterion based on our Herschel-Bulkley constitutive model. Thus the final time step criterion is given by -

$$\Delta t_{cr} = min\left(\beta \frac{\Delta x}{v_{max}}, \alpha\sqrt{\frac{4\mu}{3\rho} + \frac{\kappa}{2\rho}(J + \frac{1}{J})}, \Delta t_{prev}\right) \tag{2}$$

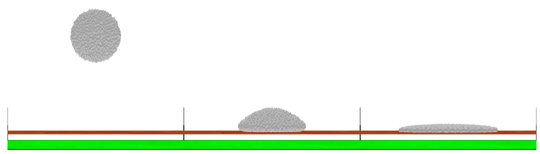


Figure 7: This figure shows a spherical cream splashing on the ground.

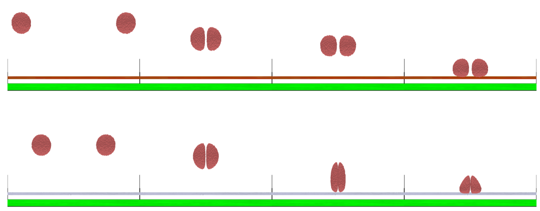


Figure 8: This figure compares collision between two spherical materials of different type. Top row contains oobleck sphere colliding while bottom row contains two pies colliding.

where Δx is the grid spacing, ρ is the density of the particle, μ is the shear modulus, κ is the bulk modulus, J is the determinant of the deformation gradient and Δt_{prev} is the last computed stable time step. α and β are the tuning parameters which were fixed at 0.1 and 0.5 respectively.

Using our custom adaptive time stepping algorithm we were able to speed on our experiments by 3-10 times.

7 RESULTS

For this rotation project, we extended the already available implementation on simulating anisotropic elasto-plastic materials. We worked on modifying the implementation to first include some different elastic models to understand the functionality of Material Point Methods (Figure - **4**). After successfully implementing the elastic models, we implemented the Herschel-Bulkley model to simulate the non-Newtonian fluids as described in Section **5**. We experimented this model on different type of fluids - specifically cream (Figure - **7**) , pie (Figure - **8**) and oobleck (Figure - **1**, **8**, **9**). We also experimented the examples with complex objects such as bunny (Figure - **5**) and dragon (Figure - **6**) and see their unique behavior when made of different type of fluids. At last to speed up the simulation time - we implemented the adaptive time stepping method. The different images showcase the results that we obtained.

8 CONCLUSION AND FUTURE WORK

During this rotation, we were able to simulate the shear thickening/shear thinning fluids. Our main focus was on getting the oobleck materials behavior as close to actual behavior. The next plan is to add ferro-magnetic properties to the oobleck material by adding ferral fluids and controlling the thus formed ferral-oobleck

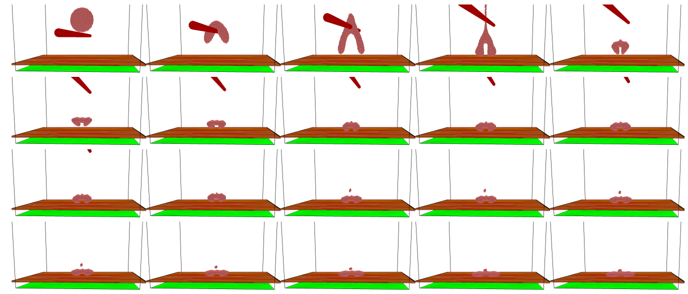


Figure 9: This figure showcases a rod cutting the oobleck ball.

with magnetic force. The interesting thing about oobleck type materials is that it behaves as a solid if subjected to strong stress. But if left free or under low stress, it flows as a liquid. The idea is to exploit this behavior of oobleck and apply the magnetic fields to control the fluid's shape and behavior.

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